



## A Review on Optimal Reinsurance under Ruin Probability Constraint

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### Öz

Bu çalışmada iflas olasılığı kısıtı altında optimal reasüransın belirlenmesine yönelik en çok tercih edilen ve kullanılan yöntemler araştırılmıştır. Bu kapsamda, iflas olasılığına bağlı optimal reasüransın incelendiği literatürdeki çalışmalar derlenerek kullanılan yöntemler sınıflandırılmıştır. Literatürdeki çalışmaların incelenmesiyle iflas olasılığı kısıtı altında optimal reasürans üzerine yapılan çalışmaların düzeltme katsayısı (adjustment coefficient), reasürans anlaşmaları, yatırım-optimizasyon teknikleri ve temettü ödemeleri olmak üzere dört temel yaklaşım altında incelendiği belirlenmiştir.

**Anahtar sözcükler:** İflas Olasılığı, Risk Kuramı, Optimal Reasürans.

### Abstract

#### *A Review on optimal reinsurance under ruin probability constraint*

The main purpose of this study is to present the commonly used and the most preferred methods on optimal reinsurance under the ruin probability constraint. In this context, we review the literature relevant to the optimal reinsurance under the ruin probability constraint and categorized current methods. The literature review on optimal reinsurance under the ruin probability constraint is mainly focus on four major aspects: reinsurance arrangement, adjustment coefficient, investment and dynamic optimization techniques, and dividend payments.

**Keywords:** Ruin Probability; Risk Theory; Optimal Reinsurance.

### 1. Introduction

Ruin theory has been one of the most important working area in actuarial science. The main aim of the research on the risk process is to indicate whether an insurance company will remain solvent or not. In addition, ruin probability is a measure that allows the insurance company to investigate whether the collected premiums are sufficient to pay the claims, starting with a certain initial surplus, on a finite time horizon.

A wide variety of models and assumptions are investigated under the consideration of the ruin theory. The classical risk models assume that the premium income has a constant rate and the aggregate claim amount has a compound Poisson process. Although the literature on the ruin probability on the classical risk model shows a variety of approaches, in the last few years there has been a growing interest on minimizing the ruin probability or maximizing the survival probability of the insurance company. An insurance company has to control its ruin probability and keep it at a minimum level to be able to sustain its existence. Previous studies indicate that by making a reinsurance arrangement, dividend payments or an investment, the ruin probability can be controlled and can be minimised [6].

Most of the literature on minimizing the ruin probability is based on reinsurance arrangements. In traditional reinsurance arrangements; excess of loss reinsurance and proportional reinsurance, the reinsurance premium is calculated according to the reinsurance level and the reinsurance loading factor. However, in a new kind of reinsurance arrangement, the initial surplus also has an essential role in determining the reinsurance premium. Several publications on deciding the optimal reinsurance level which minimise the ruin probability have appeared in recent years [6].

The classical risk process is based on initial surplus, premiums and claims. It is assumed that the surplus process starts with an initial level  $u$  and continues according to two opposing cash flows: the premium income per unit of time and the aggregate claim amount up to time  $t$ , denoted by  $S(t)$ . The insurer's surplus (or risk) process,  $(U(t))_{t \geq 0}$  is defined by

$$U(t) = u + ct - S(t).$$

The aggregate claim amount up to time  $t$ ,  $S(t)$ , is

$$S(t) = \sum_{i=1}^{N(t)} X_i.$$

where  $N(t)$  denotes the number of claims that occur in the fixed time interval  $[0, t]$ . The aggregate claim amount consists of two main components. The individual claim amounts, modelled as independent and identically distributed (i.i.d) random variables  $\{X_i\}_{i=1}^{\infty}$  with distribution function  $F(X) = \Pr(X_1 \leq x)$  such that  $F(0) = 0$  and  $X_i$  is the amount of the  $i^{th}$  claim. We use the notation  $f$  and  $m_k$  to represent the density function and  $k^{th}$  moment of  $X_1$ , respectively, and it is assumed that  $c > E[N] m_1$ .

The probability of ruin in continuous time for infinite time (ultimate ruin probability) is defined as

$$\psi(u) = \Pr(U(t) < 0 \text{ for some } t > 0).$$

$\psi(u)$  is the probability that the insurer's surplus falls below zero at some time in the future, that is claims outgo exceed the initial surplus plus premium income.

The probability of ruin in continuous and the finite time is defined as

$$\psi(u, t) = \Pr(U(t) < 0 \text{ for some } s, \quad 0 < s \leq t),$$

where  $\psi(u, t)$  is the probability that the insurer's surplus falls below zero in the finite time interval  $(0, t]$ .

This review paper is concerned with properties and application methods of optimal reinsurance under the ruin probability constraint. We aim to present a brief literature review on optimal reinsurance under the ruin probability constraint. We categorize the methods which are used in the literature to determine which techniques are essential.

## 2. Optimal Reinsurance Under The Ruin Probability Constraint

In a reinsurance arrangement, the optimal level and the type of reinsurance can be determined under a constraint such as ruin probability or risk measure. Most of the actuarial literature focus on the determination of an optimal level for a reinsurance arrangement. Many studies have been carried out on the calculation of optimal reinsurance under the ruin probability constraint. The studies that consider the optimal reinsurance level under the ruin probability constraint can be categorized according to the reinsurance principles, the adjustment coefficient, investment- dynamic optimization techniques and dividend payments. We mainly point out recent studies which discuss the optimal reinsurance under the ruin probability constraint.

### 2.1 Optimal Reinsurance By Using The Adjustment Coefficient

Previous studies indicate that the optimal reinsurance on ultimate ruin probability is mostly related with the adjustment coefficient. Thus, such as in Cramer-Lundberg approximation, the ruin probability decreases fastly when the adjustment coefficient increases. The effect of the adjustment coefficient on the ultimate ruin probability under the Cramer-Lundberg approximation is examined in various papers.

In the classical risk process,  $N(t)$  is assumed as a homogeneous Poisson process with intensity  $\lambda$ ,  $N(t) \sim \text{Poisson}(\lambda t)$ ,  $U(t)$  is called a classical risk model or Cramer-Lundberg model.

Cramer condition is found by using Esscher transform of  $F$ ,

$$\frac{\rho}{\mu} \int_0^{\infty} e^{rx} (1 - F(x)) dx = 1,$$

where  $\rho = 1/(1 + \theta)$ .

In a risk process with claims distributed as  $X \geq 0$  having  $E[X] = \mu > 0$ . The equation has a solution equal to zero and the adjustment coefficient  $R$  is the positive solution of the following equations.

$$1 + (1 + \theta)\mu_1(r) = M_X(r).$$

For the classical risk process, the adjustment coefficient can be obtained from

$$\lambda + cr = \lambda M_X(r).$$

When  $S$  is the total claims in an interval of length 1; consequently  $(c - S)$  is the profit in that interval. It is assumed that  $S$  is a compound Poisson with parameter  $\lambda$  and the moment generating function of  $S$  is  $M_S(R) = e^{\lambda(M_X(R)-1)}$ . Then  $R$  can be found as the positive root to any of the following equations [30].

$$e^{Rc} = E[e^{RS}] \quad \Longleftrightarrow \quad M_{c-S}(-R) = 1 \quad \Longleftrightarrow \quad c = \frac{1}{R} \log(M_S(R))$$

A closed form of this equation cannot be obtained. However, it can be used to derive an inequality.

For compound Poisson process the upper bound for the infinite time ruin probability yields that

$$\psi(u) \leq e^{-Ru}.$$

Cramer-Lundberg's asymptotic infinite time ruin probability formula for large value of initial

surplus  $u$  is given as

$$\psi(u) \sim C e^{-Ru},$$

where

$$C = \frac{\theta\mu}{M'_X(R) - \mu(1+\theta)}.$$

The main limitation of the Cramer-Lundberg approximation is that it requires the existence of the adjustment coefficient and takes light-tailed distributions into consideration. When the individual claim amounts have exponential distribution, this formula yields the exact infinite time ruin probability.

When the individual claim amounts are distributed exponentially with the parameter  $\alpha$ , the exact infinite time ruin probability can be obtained as

$$\psi(u) = \psi(0)e^{-Ru},$$

where  $\psi(0) = 1/(1 + \theta)$  and  $R = \alpha - \lambda/c$ .

One of the first examples of minimizing the ruin probability by using the adjustment coefficient is presented by Gerber [21]. He shows the optimal retention level for the excess of loss reinsurance. Gerber [21] assumes that the expected claim amount for the reinsurer is a function of the individual claim amount in order to maximise the adjustment coefficient  $R$ . The same results are obtained by Bowers et al. [4] who suggest to consider the reinsurance to maximise the expected utility.

Waters [47] introduces the optimal levels of the excess of loss reinsurance for a portfolio of risks in order to minimise the ruin probability. The aggregate claims are distributed as a compound Poisson and the expected value premium principle is used for the calculation of the reinsurance premiums. Waters [47] aims to find an optimal level for the excess of loss reinsurance under the minimum ultimate ruin probability. The ruin probability is minimised by using the insolvency constant (as adjustment coefficient) of the insurer and this constant is defined as a function of the excess of loss reinsurance retention level. He also investigates how this insolvency constant can be maximised.

Waters [48] extends his study according to the different individual claims and premium principles. The results suggest that the optimal retention level is obtained to satisfy the equation of  $M = R^{-1} \ln(1+\theta)$  under the assumption of the aggregate claims are compound Poisson and premiums are calculated according to the expected value premium principle. It is also featured that the adjustment coefficient is a function of the retention level in a proportional case without any restrictive assumptions on the distribution of the annual claims. Moreover he finds that in a non-proportional case, it cannot be possible to generalize the distribution of annual claims or the way in which the reinsurance premium is calculated. This approach is proposed for the proportional case.

Centeno [9] assumes that the annual claims are distributed as compound Poisson and the premiums are calculated according to the expected value principle. She implements the combinations of quota-share with the excess of loss reinsurance which is suggested by Centeno [8] under the assumption of the claims described as an ordinary renewal process.

Centeno [11] also points out an approach for the insurers' adjustment coefficient as a function of the retention levels. Centeno [11] relates the retention level to a combination of quota share with the excess of loss reinsurance in the Sparre Anderson model.

Hesselager [23] aims to find optimal reinsurance treaties in order to minimise the ruin probability. Hesselager [23] uses three different functions to identify the reinsurance arrangements and also uses Vajda's compensation functions to compare the reinsurer's adjustment coefficient of these functions.

Centeno [10] considers the optimal retention level  $M$  under the excess of loss reinsurance so that the upper bound of the probability of ruin is minimised. Centeno [10] improves Lundberg's inequality for the finite time to redefine the optimal retention limit for an excess of loss arrangement. Moreover Centeno [10] defines the optimal retention  $M$  to minimise the upper bound of the ultimate ruin probability.

Dickson and Waters [15] indicate obtaining the optimal retention level when the reinsurance loading factor depends on the retention level. Dickson and Waters [15] discuss the effect of the reinsurance both on finite and infinite time ruin probabilities. Additionally, this paper demonstrates the feasibility of the translated gamma process on the optimal retention levels.

Liang and Guo [35] mention that the insurer's surplus process is defined by Brownian motion and they propose an explicit expression in the diffusion approximation and jump-diffusion case to find an optimal strategy to minimise the ruin probability by maximizing the adjustment coefficient.

Guerra and Centeno [22] study on the optimal reinsurance policy considering both the adjustment coefficient and the expected utility. Guerra and Centeno [22] deal with the relationship between maximizing the adjustment coefficient and maximizing the expected utility of wealth.

Kolkovska [34] analyses the reinsurance treaties and the existence of the insurer's adjustment coefficient as a function of the retention levels. Kolkovska [34] proposes two different types of reinsurance treaties as a combination of excess of loss and quota-share. Moreover, Kolkovska [34] shows some results obtained from these arrangements.

Centeno and Simoes [12] present a summary of the previous studies on optimal reinsurance.

## 2.2 Optimal Reinsurance By Using Reinsurance Principles

The focus of recent studies is to construct the optimal reinsurance by reinsurance principles. Previous studies indicate that the excess of loss reinsurance and proportional reinsurance have attracted much attention on optimal reinsurance. We started by investigating the properties of these reinsurance arrangements.

Under an excess of loss reinsurance arrangement, the insurer and the reinsurer's expected individual claim amounts are calculated according to a constant retention level  $M$ .

When a claim  $X$  occurs, the insurer pays  $Y = \min(X, M)$  and the reinsurer pays  $Z = \max(0, X - M)$  with  $X = Y + Z$ . Hence, the distribution function of  $Y$ ,  $F_Y(x)$ , is

$$F_Y(x) = \begin{cases} F_X(x) & \text{for } x < M, \\ 1 & \text{for } x \geq M, \end{cases}$$

and the moments of  $Y$  is

$$E[Y^n] = \int_0^M x^n f(x) dx + M^n (1 - F(M)).$$

Similarly, the moments of  $Z$  are

$$E[Z^n] = \int_M^\infty (x - M)^n f(x) dx.$$

Under the proportional reinsurance, a fixed proportion is used instead of the retention level  $M$  in the excess of loss reinsurance. The insurer's liability is limited to the proportion  $p$  of each claim; conversely, the reinsurer pays proportion  $1-p$  of each claim. Therefore, the reinsurance premium is calculated depending on this same proportion.

Let  $Y$  and  $Z$  represent the part of the claim paid by the insurer and the reinsurer, respectively. Under the proportional reinsurance,  $Y = pX$  and  $Z = (1 - p)X$  with  $X = Y + Z$ .

The distribution function of  $Y$ ,  $F_Y(x)$ , and the density function  $f_Y(x)$ , are obtained as  $F_X(x/p)$  and  $\frac{1}{p}f_X(x/p)$  respectively.

One of the first examples of optimal reinsurance by reinsurance principle is presented by De Finetti [13]. This paper clearly shows that the insurer's risk such as the variance of the profit and ruin probability can be reduced by using any reinsurance arrangement. He determines the optimal retention level for a non-life insurance portfolio under a constraint of a minimum variance of the insurer's profit for a fixed expected profit. De Finetti [13] shows how optimal levels should be calculated for both excess of loss and the proportional reinsurance under the minimum variance criterion for the insurer's expected profit. De Finetti [13] points that the retention level is proportional to the insurance loading factor and inversely proportional to the variance of the risk. Further details and proofs can be found in Bühlmann [5].

Dickson and Waters [16] consider the optimal reinsurance levels under the criterion of minimizing the finite time ruin probability for both discrete and continuous time in a non-life insurance portfolio. Dickson and Waters [16] focus on minimizing the ruin probability instead of the variance criterion in De Finetti's approach [13] and they describe a constraint as the expected profit per unit time is greater than zero or equal to it. In the paper, it is assumed that the aggregate claims process can be approximated by a translated gamma process.

Ignatov et al. [27] derive a formula for the expected profit under the probability of survival of the insurer. They consider estimating the optimal levels by using an optimality criterion for the expected profit formula from both the insurer's and the reinsurer's points of view. The optimality is explained as the levels that maximise the joint survival probability up to a finite time horizon of the cedent and the reinsurer.

Kaluszka [33] proposes the optimal reinsurance to minimise the ruin probability for the truncated stop loss reinsurance. He applies the truncated stop loss contracts for different pricing rules such as economic principle, generalized zero-utility principle, Esscher principle and mean-variance principle. Kaluszka [33] indicates that the result is also valid for models not involving the ruin probability such as the expected utility, the Kahneman-Tversky value function of the cedent, the variance of the cedents cover and the dividend payment.

Dickson and Waters [18] minimise the ruin probability by using the retention levels in accordance with a dynamic reinsurance strategy. They derive a formula for the finite time ruin probability for discrete and continuous time by using the Bellman optimality principle. In addition, they show how this formula can be used both by approximating the compound Poisson aggregate claims distribution with a translated gamma distribution, and by approximating a compound Poisson process with a translated gamma process, respectively.

Kaishev and Dimitrova [31] generalize a joint survival optimal reinsurance model for the excess of loss reinsurance. They assume that the individual claim amounts are modelled by continuous dependent random variables with a joint distribution. Based on maximizing the probability of joint survival and level of premium income, the optimal levels of the excess of loss reinsurance are obtained.

Nie et al. [40] propose a different kind of reinsurance arrangement in which the reinsurer's payments are bounded above by a fixed level. According to Nie et al. [40], whenever the surplus falls between zero and this fixed level, the reinsurance company makes an additional payment in such a way that the surplus process returns to the fixed level. The reinsurance premium is also calculated according to these capital injections. The optimal initial surplus and the fixed reinsurance level are calculated so that the ultimate ruin probability is minimised.

### 2.3 Optimal Reinsurance By Using Investment And Optimization Techniques

Several publications have appeared in recent years documenting on constructing optimal reinsurance by using investment and optimization techniques.

Hipp and Plum [24] present that the ruin probability can be minimised by investment in a risky asset in the classical Cramer-Lundberg model. In this model, the interest rate of the bond is assumed to be zero. They use the Bellman equation to determine the optimal strategy and give explicit solution for the exponential and Pareto claim size. They also investigate the effect of the optimal investment strategy and ruin probability according to the changes in some factors such as stock volatility.

Liu and Yang [37] develop the model which is proposed by Hipp and Plum [24] with respect to a non-zero interest rate. They assume that the premiums are calculated by a constant rate and investment in a risky asset. Liu and Yang [37] use Hamilton-Jacobi- Bellman equation to determine the optimal investment strategy and compare these strategies under the various claim size distributions. Liu and Yang [37] further assume that the fraction of an amount available for investment in a risky asset is proportional to insurer's surplus.

Schmidli [41] examines the dynamic proportional reinsurance to minimise the ruin probability. He develops an optimal unlimited proportional reinsurance strategy. Schmidli [41] also compares the results of both the diffusion and Cramer-Lundberg cases. He uses the Hamilton-Jacobi-Bellman equation and presents a proof of the existency of the solution.

Schmidli [42] suggests the optimal investment and reinsurance strategies to minimise the ruin probability. A key limitation of this research is the assumption of investing in only a risky assets which are modelled by Black-Scholes formula. The optimal levels of investment and reinsurance are obtained by using Hamilton-Jacobi-Bellman approach. Schmidli [42] concludes that the investment and reinsurance decrease the ruin probability for larger initial surplus under the Pareto claim sizes. Taksar and Markussen [45] develop this approach in regards to the diffusion model.

Hipp and Vogt [25] examine the dynamic excess of loss reinsurance. They suggest a dynamic unlimited excess of loss reinsurance strategy to minimise the infinite time ruin probability. Hipp and Vogt [25] use the classical Hamilton-Jacobi-Bellman equation to maximise the survival probability for an optimal strategy and give numerical examples for three different claim size distributions namely exponential, shifted exponential, and Pareto.

Castillo and Parrocha [7] describe the stochastic control methods to determine the optimal investment strategy which minimises the ruin probability. In this context, the existence of a solution of Hamilton-Jacobi-Bellman equation is proved and numerical algorithm is given. They consider an insurance business with a fixed amount of investment in a portfolio which consists of one risky and one risk free asset.

Gaier and Grandits [19] develop an optimal investment with positive interest force to minimise the ruin probability. They analyse the ruin probability for the proportional investment for the claims distributed by regularly varying tails which is one of subclass of heavy tailed.

Hipp [26] briefly reviews the methods for the optimal investment problem by using stochastic control theory on minimizing the ruin.

Schmidli [43] uses both investment and proportional reinsurance in a classical risk process and defines the Cramer-Lundberg approximation for the ruin probability. It is assumed that investment into a risky asset is modelled by a Brownian motion. Under the assumption of nonexistence of the exponential moments, the Lundberg exponent is redefined and an upper bound for the ruin probability is calculated by using Hamilton-Jacobi-Bellman equation.

Irgens and Paulsen [28] discuss the maximization of the expected utility of the asset of an insurance company under the reinsurance and investment constraints in a diffusion-perturbated classical risk model. The risky or non-risky assets are used for the investment and some of the company's risk is covered by the proportional reinsurance and the remaining part is covered by the excess of loss reinsurance. They use three different utility functions for the expected utility: exponential, logarithmic and power utility with termination at ruin. A key limitation of this paper is that the insurance company cannot invest in a risk-free and a risky asset at the same time.

Ma et. al [38] concentrate on the possibility of investing in a risk-free asset to find the optimal proportional reinsurance strategy in order to minimise the probability of ruin in the classical risk model. They conclude that optimal proportional is valid to minimise the ruin probability or to maximise the survival probability.

Kasumo [32] and Joseph [29] determine the role of investment on minimizing the ruin probability. While Kasumo [32] studies the proportional reinsurance case, Joseph [29] analyses the excess of loss reinsurance. The risk process is assumed as a perturbated-diffusion risk model and investment is modelled as a Black-Scholes model in both studies. In addition, the optimal investment is examined with Hamilton-Jacobi-Bellman approach and Volterra Integro Differential equation which is transformed into a linear Volterra Integral Equation of second order is proved.

The previous studies show that Hamilton-Jacobi-Bellman equation is necessary in determination of the optimal investment and reinsurance.

## 2.4 Optimal Reinsurance By Using Dividend Payments

In the literature there are also some papers on constructing optimal reinsurance using dividend payments. The optimal dividend problem is suggested by De Finetti [14]. He proposes a strategy that "maximises the expected value of discounted dividend payments until the time of ruin". This problem is firstly solved by Gerber [20] for the classical Cramer-Lundberg model.

Dickson and Waters [17] extend De Finetti [14]'s approach by allowing the process to continue after the ruin. They examine the optimal dividend strategy which maximises the expected discounted future dividends by using barrier level in modified surplus process. They assume that when the surplus reaches the level  $b$ , the premium income cannot be used in the surplus process and the premium income directly goes into paying dividends to shareholders. Hence, the premium income is only enough to pay dividend and ruin occurs when the modified surplus process falls below zero.

Baurle [2] concentrates on the reinsurance policy in the presence of dividend strategies for piecewise-deterministic Markov processes.

Kaluszka [33] examines the optimal reinsurance level on a truncated stop loss reinsurance and provides the optimality according to several models with regard to maximizing the expected dividend payment. He assumes that the insurer pay dividends if the insurer's gain exceeds a level such as expenses plus profit after the reinsurance.

Azcue and Muler [1] find a dynamic strategy which contains both the reinsurance policy and the dividend distribution strategy that maximises the cumulative expected discounted dividend payouts for Cramer-Lundberg model.



Schmidli [44] focuses on two main topics to minimise the ruin probability by using the proportional reinsurance and to maximise the expected discounted dividend payout. On minimizing the ruin probability part, he proves the existency and uniqueness of the Hamilton- Jacobi-Bellman equation and verification theorem. The Hamilton-Jacobi-Bellman equation is used on maximization of the expected discounted dividend payout and obtaining the optimal strategy.

Thonhauser and Albrecher [46] suggest that a value function which considers both expected dividends and the time value of ruin to find the optimal dividend payment strategy for maximizing the expected value of the discounted dividend payments until ruin. The optimality of barrier strategy is obtained for both the diffusion model and the Cramer-Lundberg model with exponential claim sizes.

Beveridge et al. [3] examine to find out whether or not the expected present value of net income to shareholders can be increased by reinsurance when there is a constant dividend barrier. In addition, they use both De Vylder's approximation and Dickson and Waters's discrete time model [16] to identify an optimal reinsurance strategy.

Liang and Yao [36] present a nonlinear-singular stochastic optimal control on reinsurance rate and dividend payout process under the ruin probability constraints. They aim to maximise the expected present value of the dividend payments until the time of ruin. They also discuss solving the optimality problems such as optimal retention ratio, dividend payout level, optimal return function and optimal control strategy of the insurance company.

Meng and Siu [39] describe the optimal reinsurance and dividend strategy under the consideration of reinvestment or retained earnings. Meng and Siu [39] aim to maximise the difference between the expected discounted dividends minus the expected discounted reinvestment until the time of ruin. In this context, the optimal reinsurance strategy is obtained under the excess of loss reinsurance.

Zhou and Yuen [50] concentrate on the optimal dividend strategy with the proportional reinsurance and capital injection in a large portfolio. Zhou and Yuen [50] study on maximizing differences between the expected value of the discounted dividend payments and the discounted costs of capital injection. Optimal strategy is obtained in consideration of the proportional reinsurance, variance premium principle, and capital injection at ruin time.

Wu [49] focuses on maximizing the difference between the cumulative expected discounted dividend and the penalized discounted capital injections until the time of ruin. It is assumed that the company controls the dividend payments to the shareholders as well as capital injection. He assumes that optimal combination reinsurance strategy consists of only the excess of loss reinsurance.

### 3. Conclusions

In this study, we have focused on reviewing the recent studies which discuss the optimal reinsurance under the ruin probability constraint. From the research that has been conducted, it is possible to conclude that the optimal reinsurance under the ruin probability constraint can be categorized into four topics: the adjustment coefficient, the reinsurance principles, investment with dynamic optimization techniques and dividend payments. Moreover, the authors' attention is concentrated not only on finding the optimal reinsurance in order to minimise the ruin probability but also on maximizing some criteria such as investment earnings, dividend payments, expected utility or expected profit.

However, to the authors' best knowledge, very few publications exist in the literature that discusses more than one measure. In addition, most of the previous studies on optimal reinsurance under the ruin probability constraints do not take multiple criteria into account. The literature review shows that most of the studies focus on optimal reinsurance based only on a single constraint and the optimal reinsurance strategy change under different constraints.

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